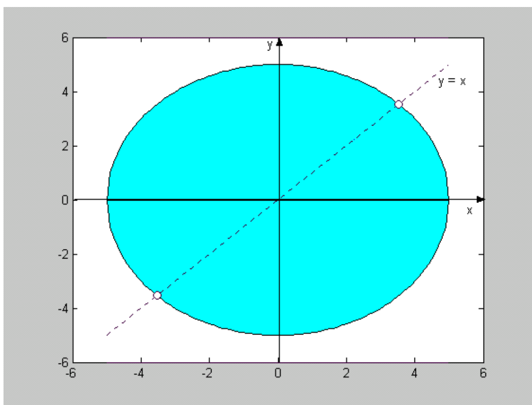
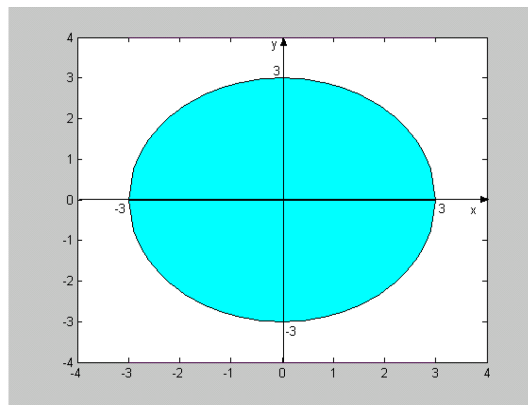
1. a) campo escalar $Dom \subseteq R^3$ $Codom \subseteq R$
   b) función vectorial $Dom \subseteq R$ $Codom \subseteq R^2$
   c) función vectorial $Dom \subseteq R$ $Codom \subseteq R^3$
   d) campo vectorial $Dom \subseteq R^2$ $Codom \subseteq R^2$
   e) campo vectorial $Dom \subseteq R^3$ $Codom \subseteq R^3$
   f) campo escalar $Dom \subseteq R^3$ $Codom \subseteq R$

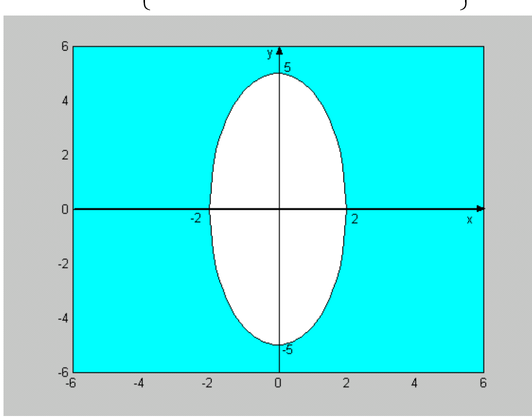
3. a) $DomF = R$
   b) $DomF = \{t \in R \,/\, t > 0\}$
   c) $DomF = \{t \in R \,/\, 3 - t \geq 0 \wedge t - 4 \geq 0 \wedge t^2 + 1 \neq 0\} = \varnothing$
   d) $DomF = \{t \in R \,/\, -2 < t \leq 2 \wedge t \neq 1 \wedge t \neq 0\}$

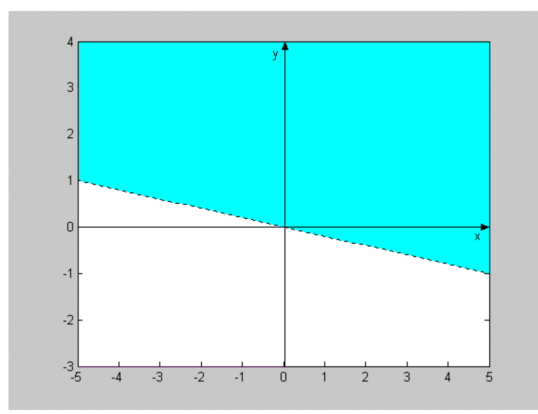
4. a) $Dom_f = \{(x;y) \in R^2 \,/\, x^2 + y^2 \leq 2 \wedge x \neq y\}$ b) $Dom_f = \{(x;y) \in R^2 \,/\, x^2 + y^2 \leq 9\}$



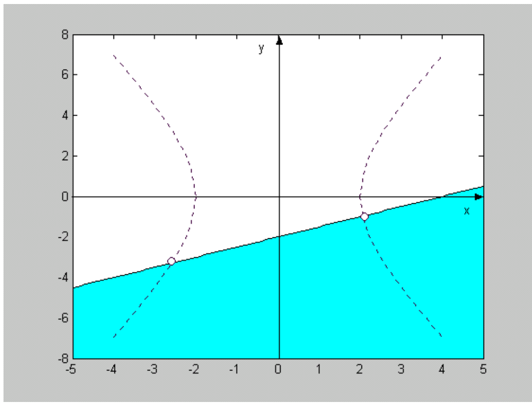


c) $Dom_f = \left\{(x;y) \in R^2 \,/\, \dfrac{x^2}{4} + \dfrac{y^2}{25} \geq 1\right\}$ d) $Dom_f = \{(x;y) \in R^2 \,/\, y > -1/5\, x\}$

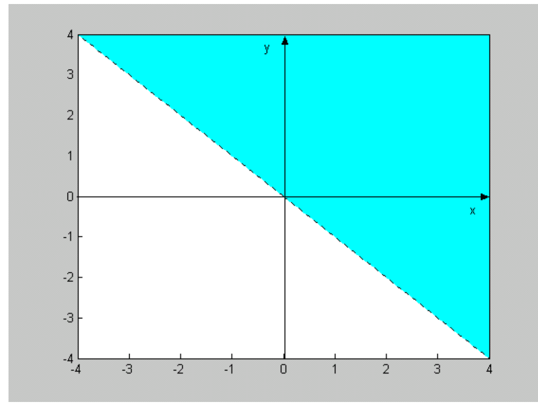




e) $Dom_f = \left\{ (x;y) \in R^2 \,/\, y \le -2 + \frac{1}{2}x \wedge \frac{x^2}{4} - \frac{y^2}{16} \ne 1 \right\}$



f) $Dom_f = \{(x,y) \in R^2 \,/\, y > -x\}$



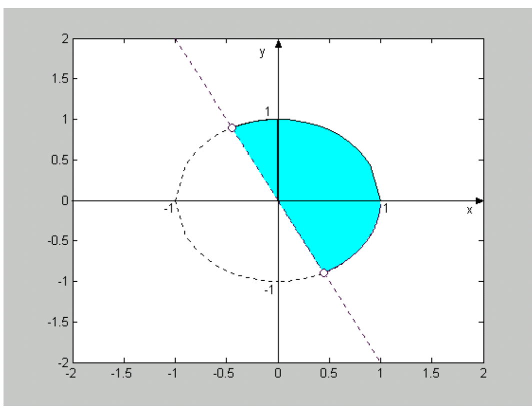
g) $Dom_f = R^2$

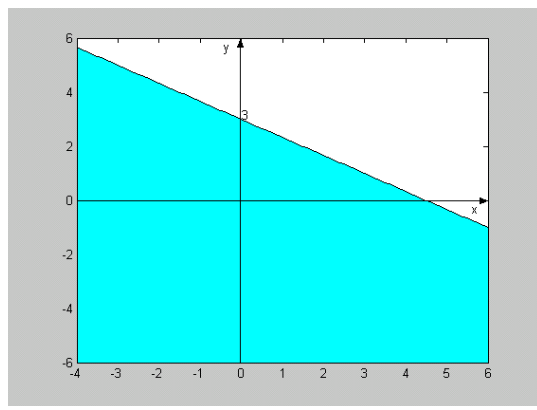
h) $Dom_f = \{(x,y) \in R^2 \,/\, y \ne 0\}$

i) $Dom_f = \{(x,y) \in R^2 \,/\, y \ne 4 - x\}$

j) $Dom_f = \{(x,y) \in R^2 \,/\, x^2 + y^2 \le 1 \wedge y > -2x\}$

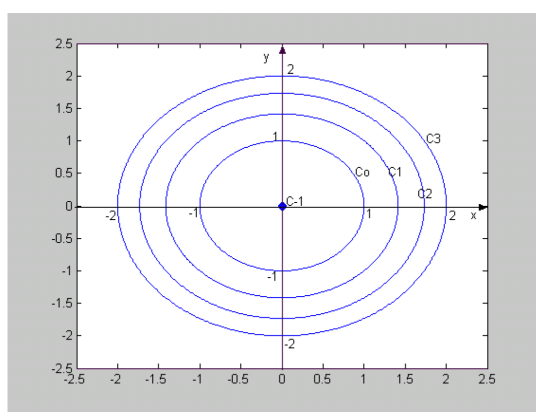
k) $Dom_f = \{(x,y) \in R^2 \,/\, y \le -2/3\, x + 3\}$





5. a) $Dom_F = \{(x,y) \in R^2 \,/\, y \ne -x\}$
   b) $Dom_F = \{(x,y) \in R^2 \,/\, y < x + 3\}$
   c) $Dom_F = \{(x,y) \in R^2 \,/\, x^2 + y^2 > 9 \wedge y > -1/2\, x \wedge y \ne -2x\}$

6. a) $C_{-2} = \{(x,y) \in D_f \,/\, x^2 + y^2 = -1\} = \varnothing$
   $C_{-1} = \{(x,y) \in D_f \,/\, x^2 + y^2 = 0\} = \{(0;0)\}$
   $C_0 = \{(x,y) \in D_f \,/\, x^2 + y^2 = 1\}$
   $C_1 = \{(x,y) \in D_f \,/\, x^2 + y^2 = 2\}$
   $C_2 = \{(x,y) \in D_f \,/\, x^2 + y^2 = 3\}$
   $C_3 = \{(x,y) \in D_f \,/\, x^2 + y^2 = 4\}$



b) $C_{-2} = \{(x;y) \in D_f \ / \ y = -\dfrac{1}{x}\}$

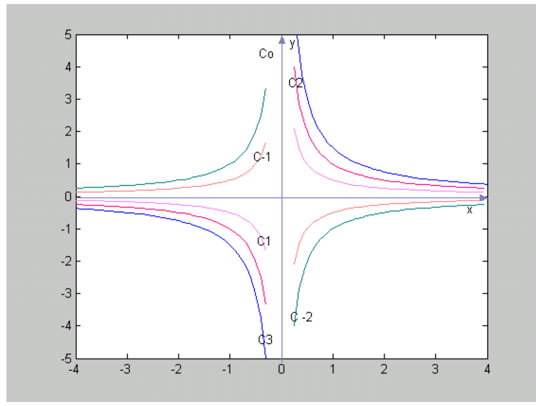
$C_{-1} = \{(x;y) \in D_f \ / \ y = -\dfrac{1}{2x}\}$

$C_0 = \{(x;y) \in D_f \ / \ 2xy \neq 0\}$

$C_1 = \{(x;y) \in D_f \ / \ y = \dfrac{1}{2x}\}$

$C_2 = \{(x;y) \in D_f \ / \ y = \dfrac{1}{x}\}$

$C_3 = \{(x;y) \in D_f \ / \ y = \dfrac{3}{2x}\}$



c) $C_{-2} = \{(x;y) \in D_f \ / \ y = -1/2x^2 - 2\}$

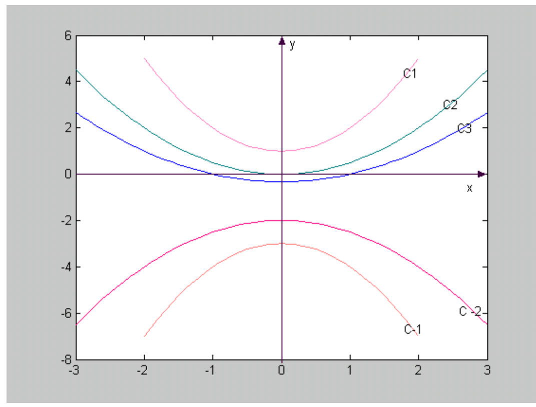
$C_{-1} = \{(x;y) \in D_f \ / \ y = -x^2 - 3\}$

$C_0 = \varnothing$

$C_1 = \{(x;y) \in D_f \ / \ y = x^2 + 1\}$

$C_2 = \{(x;y) \in D_f \ / \ y = 1/2x^2\}$

$C_3 = \{(x;y) \in D_f \ / \ y = 1/3x^2 - 1/3\}$



d) $C_{-2} = \{(x;y) \in D_f \ / \ y = -2 - x\}$

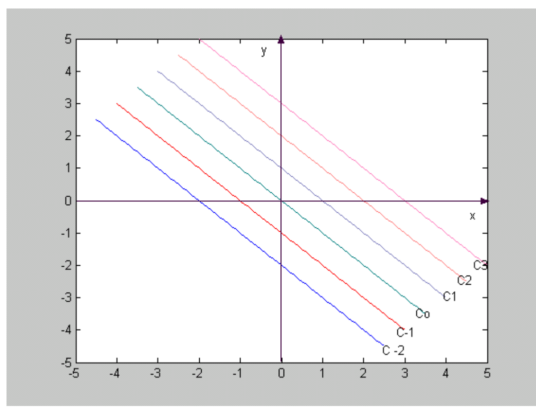
$C_{-1} = \{(x;y) \in D_f \ / \ y = -1 - x\}$

$C_0 = \{(x;y) \in D_f \ / \ y = -x\}$

$C_1 = \{(x;y) \in D_f \ / \ y = 1 - x\}$

$C_2 = \{(x;y) \in D_f \ / \ y = 2 - x\}$

$C_3 = \{(x;y) \in D_f \ / \ y = 3 - x\}$



e) $C_{-2} = \{(x;y) \in D_f \ / \ y = -1/2 - x\}$

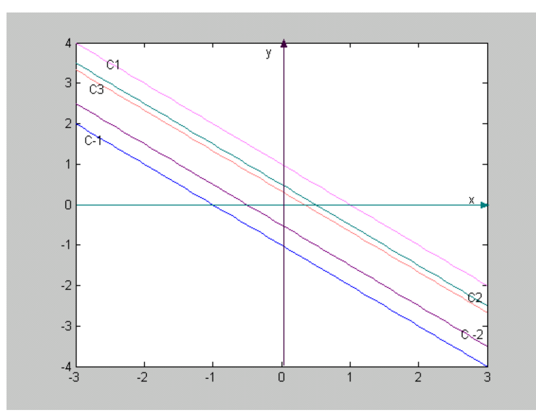
$C_{-1} = \{(x;y) \in D_f \ / \ y = -1 - x\}$

$C_0 = \varnothing$

$C_1 = \{(x;y) \in D_f \ / \ y = 1 - x\}$

$C_2 = \{(x;y) \in D_f \ / \ y = 1/2 - x\}$

$C_3 = \{(x;y) \in D_f \ / \ y = 1/3 - x\}$



g) $C_{-2} = \varnothing$

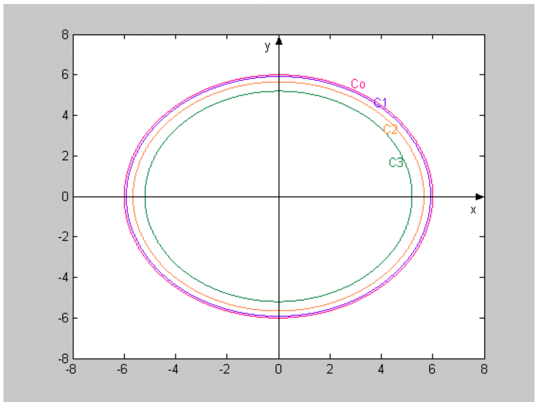
$C_{-1} = \varnothing$

$C_0 = \{(x;y) \in D_f \; / \; x^2 + y^2 = 36\}$

$C_1 = \{(x;y) \in D_f \; / \; x^2 + y^2 = 35\}$

$C_2 = \{(x;y) \in D_f \; / \; x^2 + y^2 = 32\}$

$C_3 = \{(x;y) \in D_f \; / \; x^2 + y^2 = 27\}$



h) $C_{-2} = \varnothing$ $\quad C_1 = \varnothing$

$C_{-1} = R^2$ $\quad C_2 = \varnothing$

$C_0 = \varnothing$ $\quad C_3 = \varnothing$

i) $C_{-2} = \varnothing$

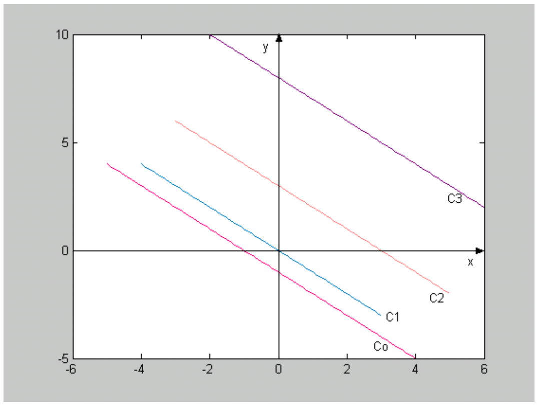
$C_{-1} = \varnothing$

$C_0 = \{(x;y) \in D_f \; / \; y = -x - 1\}$

$C_1 = \{(x;y) \in D_f \; / \; y = -x\}$

$C_2 = \{(x;y) \in D_f \; / \; y = -x + 3\}$

$C_3 = \{(x;y) \in D_f \; / \; y = -x + 8\}$



j) $C_{-2} = \{(x;y) \in D_f \; / \; -\dfrac{x^2}{2} + \dfrac{y^2}{2} = 1\}$

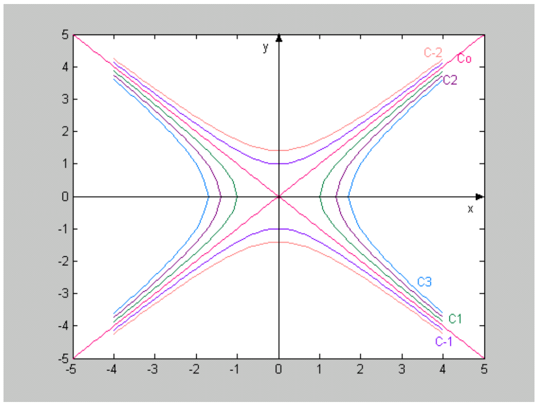
$C_{-1} = \{(x;y) \in D_f \; / \; -x^2 + y^2 = 1\}$

$C_0 = \{(x;y) \in D_f \; / \; y = x \;\vee\; y = -x\}$

$C_1 = \{(x;y) \in D_f \; / \; x^2 - y^2 = 1\}$

$C_2 = \{(x;y) \in D_f \; / \; \dfrac{x^2}{2} - \dfrac{y^2}{2} = 1\}$

$C_3 = \{(x;y) \in D_f \; / \; \dfrac{x^2}{3} - \dfrac{y^2}{3} = 1\}$



7. a) 1) $f_x^{'}(2;-3) = -9, \quad f_y^{'}(2;-3) = 23$

2) $f_x^{'}(3;-2) = 22, \quad f_y^{'}(3;-2) = -10$

3) $f_x^{'}(1;3) = 6, \quad f_y^{'}(1;3) = 3$

b) 1) $f_x^{'}(x;y) = 4xy - 5y$ $\qquad f_y^{'}(x;y) = 2x^2 - 5x$

2) $f'_x(x;y) = 6x - 2y$ $\quad f'_y(x;y) = -2x + 2y$

3) $f'_x(x;y) = 6$ $\quad f'_y(x;y) = 3$

8. 1) $\nabla f(2;-3) = (-9;\ 23)$ 2) $\nabla f(3;-2) = (22;\ 10)$ 3) $\nabla f(1;3) = (6;\ 3)$

9. a) $f'_x(x;y) = \dfrac{3}{2(3x-y)\sqrt{\ln(3x-y)}}$ $\quad f'_y(x;y) = -\dfrac{1}{2(3x-y)\sqrt{\ln(3x-y)}}$

b) $f'_x(x;y) = 2xe^{x^2}$ $\quad f'_y(x;y) = -\dfrac{3}{2}y^{1/2}$

c) $f'_x(x;y) = \dfrac{(x-y)(x+3y)}{(x+y)^2}$ $\quad f'_y(x;y) = -\dfrac{(x-y)(3x+y)}{(x+y)^2}$

d) $f'_x(x;y) = \dfrac{y}{2}e^{\frac{xy}{2}}$ $\quad f'_y(x;y) = \dfrac{x}{2}e^{\frac{xy}{2}}$

e) $f'_x(x;y) = \dfrac{x}{\sqrt{x^2+y^2}}$ $\quad f'_y(x;y) = 8y + \dfrac{y}{\sqrt{x^2+y^2}}$

f) $f'_x(x;y) = \dfrac{\sqrt{y^2-x^2} + \dfrac{x^2+xy}{\sqrt{y^2-x^2}}}{y^2-x^2}$ $\quad f'_y(x;y) = \dfrac{\sqrt{y^2-x^2} + \dfrac{xy-y^2}{\sqrt{y^2-x^2}}}{y^2-x^2}$

g) $f'_x(x;y;z) = -5x(x^2+y^2+z^2)^{-3/2}$ $\quad f'_y(x;y;z) = -5y(x^2+y^2+z^2)^{-3/2}$

$f'_z(x;y;z) = -5z(x^2+y^2+z^2)^{-3/2}$

h) $f'_x(x;y;z) = \dfrac{y}{2z}e^{\frac{xy}{z}}$ $\quad f'_y(x;y;z) = \dfrac{x}{2z}e^{\frac{xy}{z}}$

$f'_z(x;y;z) = -\dfrac{xy}{2z^2}e^{\frac{xy}{z}}$

i) $f'_x(x;y;z) = 4yz + \dfrac{z}{x}$ $\quad f'_y(x;y;z) = 4xz + \dfrac{z}{y}$

$f'_z(x;y;z) = 4yx + \ln(2xy)$

j) $f'_x(x;y) = e^{\frac{y}{x}}\left(-\dfrac{y}{x^2}\right)\ln\left(\dfrac{x^2}{y}\right) + \dfrac{2}{x}e^{\frac{y}{x}}$ $\quad f'_y(x;y) = \dfrac{e^{\frac{y}{x}}}{x}\ln\left(\dfrac{x^2}{y}\right) - \dfrac{e^{\frac{y}{x}}}{y}$

10. a) $\nabla f(2;2) = (2;\ 4)$ b) $\nabla f(-2;3) = (2;\ -5)$ c) $\nabla f(1;2) = (4;\ 5)$ d) $\nabla f(1;1;0) = (2;\ 2;\ 0)$

12. a) 1) $\Delta f = -0{,}98$ 2) $\Delta f = -0{,}40951$ 3) $\Delta f = -0{,}3$ 4) $\Delta f = 0{,}93209$

13. a) $df(x;y) = (2x + 3x^2y^3\cos(x^3y))\,\Delta x + (2y\,\mathrm{sen}(x^3y) + x^3y^2\cos(x^3y))\,\Delta y$

b) $df(x;y) == \left(\dfrac{1}{y^2} + \dfrac{1}{x}\right)\Delta x + \left(-\dfrac{2x}{y^3} + \dfrac{1}{y}\right)\Delta y$

c) $df(x;y) = \left(y^3 \cos(xy^2) + \frac{1}{y} x^{\frac{1}{y}-1} \right) \Delta x + \left(sen(xy^2) + 2xy^2 \cos(xy^2) - \frac{x^{\frac{1}{y}}}{y^2} \ln x \right) \Delta y$

d) $df(x;y) = (\frac{1}{x} + e^y)\, \Delta x + (1 + x\, e^y)\, \Delta y$

14. a) 1) $z_t = -1$ 2) $z_t = -2 + 2(x-1) + 2(y+1)$ 3) $z_t = 9/5 + 48/125\,(x-3) - 36/125\,(y-4)$
    b) $f(1{,}01;\ 0{,}98) \cong -1$ $g(1{,}01;-0{,}98) \cong -1{,}94$ $h(2{,}9;\ 4{,}01) \cong 1{,}75872$

15. a) $(1{,}02)^{3{,}03} \cong 1{,}06$ b) $\sqrt{(3{,}3)^2 + 2(2{,}1)^3} \cong 5{,}42$

16. a) $x^y \cong 1 + 2(x-1) + 1/2\ (\ 2(x-1)^2 + 2(x-1)(y-2)\ )$
    b) $(x+2)^{\,y-1} \cong 1 + \ln 2(y-1) + 1/2\ (\ x(y-1) + \ln^2 2\ (y-1)^2)$
    c) $y \ln x \cong 2(x-1) + 1/2\ (\ -2(x-1)^2 + 2(x-1)(y-2)\ )$

17. a) $\ln(1 + xy) \cong xy$
    b) $e^{\,x+y} \cong 1 + x + y + 1/2\ (\ x^2 + 2xy + y^2)\ )$
    c) $sen(x+y) \cong x + y$

18. a) $P(x;y) = 1 + 3(x-1) - (y-1) + 1/2\ (\ 6(x-1)^2 - 10(x-1)(y-1) + 2(y-1)^2\ )$
    b) $P(x;y) = 1 + 3(x-1) - (y-1) + 1/2\ (\ 6(x-1)^2 - 10(x-1)(y-1) + 2(y-1)^2\ )$
    c) $P(x;y) = 3x + 1/2\ (\ -106x^2 + 2x(y-3)\ )$

21. $(0{,}87)^{3{,}02} \cong 0{,}6581$

22. $\sqrt{1{,}03}\ \sqrt[3]{0{,}95} \cong 0{,}99769306$

23. a) 0;0;0) es punto de ensilladura.
    $f(1/2;\ -1/4) = -1/48$ es un mínimo relativo
    b) $f(1;\ -2) = 0$ es un mínimo relativo
    c) P = (0;0) es punto crítico
    d) $f(1/\sqrt[4]{3}\ ;1/\sqrt[4]{3}\ )$ es un mínimo relativo
    $(1/\sqrt[4]{3}\ ;-1/\sqrt[4]{3}\ ; f(1/\sqrt[4]{3}\ ;-1/\sqrt[4]{3}\ ))$ y $(-1/\sqrt[4]{3}\ ;1/\sqrt[4]{3}\ ; f(-1/\sqrt[4]{3}\ ;1/\sqrt[4]{3}\ ))$ son puntos de ensilladura
    $f(-1/\sqrt[4]{3}\ ;-1/\sqrt[4]{3}\ )$ es un máximo relativo
    e) (0;1;0) y (0;-1;0) son puntos de ensilladura
    f) $f(0;0) = 1$ es un máximo relativo
    g) P = (1;0) y P = (-1;0) son puntos críticos
    h) P = (2;2) es punto crítico
    i) $f(1;0) = 6$ es un máximo relativo
    $f(2;\ 1) = 4$ es un mínimo relativo
    (1;1;5) y (2;0;5) son puntos de ensilladura

24. deberá fabricar 8 docenas de la primera clase y 5 docenas de la segunda clase

25. $x = 24$, $y = 15$

26. a) $f(2;2) = 4$ es un máximo relativo condicionado
    b) $f(0;1) = 1/2$ y $f(0;-1) = 1/2$ son mínimos relativos condicionados
    $f(\sqrt{2};0) = 1$ y $f(-\sqrt{2};0) = 1$ son máximos relativos condicionados
    c) $f(6;9) = 612$ es un mínimo relativo condicionado
    d) $f(1/\sqrt{2};1/\sqrt{2}) = 2/\sqrt{2}$ es un máximo relativo condicionado
    $f(-1/\sqrt{2};-1/\sqrt{2}) = -2/\sqrt{2}$ es un mínimo relativo condicionado

27. $h(x;y) = e^{-2(x^3+xy)}sen(x^3+xy)$

28. $H(t) = \left(sen(t^{3/2}) + \ln t; \dfrac{t^{3/2}}{\ln t}; \sqrt{t}\right)$

29. $H(x;y) = \left(\sqrt{\dfrac{x}{y}}; \dfrac{1}{\ln\left(\dfrac{x}{y}\right)}\right)$

30. $H(x;y;z) = ((2x+y+z)^2 ; 4x+2y+2z ; 2x+y+z-1)$

31. a) $z'(t) = sen(t^2)\cos t + sen t \cos(t^2)\, 2t$
    b) $z'(t) = \left(sen(2^{\ln t})(t^2+2)^{sen(2^{\ln t -1})}\right)2t + \left((t^2+2)^{sen(2^{\ln t})}\ln(t^2+2)\cos(2^{\ln t})\right)\left(\dfrac{2^{\ln t}\ln 2}{t}\right)$

32. $h'(t) = 5 + 10t$

33. $h'(-3) = 1863$

34. $h'(2) = \ln 2 - 1/4$

35. $\dfrac{\partial h}{\partial u}(2;3) = 15 - 45\sqrt{6}$ $\qquad$ $\dfrac{\partial h}{\partial v}(2;3) = 50 - 30\sqrt{6}$

36. a) $\dfrac{\partial h}{\partial s} = 3s^2 - 10st + 5t^2 + t$ $\qquad$ $\dfrac{\partial h}{\partial t} = -5s^2 + 10st - 3t^2 + s$
    b) $\dfrac{\partial h}{\partial s} = 300s + 120t + 15s^2 + 14st - 10t^2s - 7t^3 + 2t^2$
    $\dfrac{\partial h}{\partial t} = 120s + 48t + 7s^2 + 4st - 21t^2s - 8t^3 - 10ts^2$

37. $F'_z(1;1;0) = 0$, entonces no se verifica el Teorema de Cauchy-Dini

38. b) $z'_x(1;-2) = -5/3$ $z'_y(1;-2) = -3/13$

39. $x'_y(1;2) = -11/4$

40. a) $x'_y = -\dfrac{xe^{xy}}{ye^{xy} + senx - 3}$ $x'_z = -\dfrac{6z-4}{ye^{xy} + senx - 3}$

$y'_x = -\dfrac{ye^{xy} + senx - 3}{xe^{xy}}$ $y'_z = -\dfrac{6z-4}{xe^{xy}}$

$z'_x = -\dfrac{ye^{xy} + senx - 3}{6z-4}$ $z'_y = -\dfrac{xe^{xy}}{6z-4}$

b) $x'_y = -\dfrac{4xy + xe^y + e^x - xz}{2y^2 + e^y + ye^x - yz}$ $x'_z = \dfrac{xy}{2y^2 + e^y + ye^x - yz}$

$y'_x = -\dfrac{2y^2 + e^y + ye^x - yz}{4xy + xe^y + e^x - xz}$ $y'_z = \dfrac{xy}{4xy + xe^y + e^x - xz}$

$z'_x = \dfrac{2y^2 + e^y + ye^x - yz}{xy}$ $z'_y = \dfrac{4xy + xe^y + e^x - xz}{xy}$

c) $x'_y = -3$ $x'_z = -2 + \dfrac{1}{z}$

$y'_x = -1/3$ $y'_z = -\dfrac{2 - \frac{1}{z}}{3}$

$z'_x = -\dfrac{1}{2 - \frac{1}{z}}$ $z'_x = -\dfrac{3}{2 - \frac{1}{z}}$

d) $x'_y = \dfrac{y-x}{y}$ $x'_z = \dfrac{z}{y}$

$y'_x = \dfrac{y}{y-x}$ $y'_z = \dfrac{z}{y-x}$

$z'_x = \dfrac{y}{z}$ $z'_y = \dfrac{x-y}{z}$

41. $dz(2;0) = 0$

42. 1) función homogénea de grado 2 $\forall t \in R$
2) función positivamente homogénea de grado 1
3) no es función homogénea
4) función homogénea de grado 0 $\forall t \neq 0$
5) función homogénea de grado 3 $\forall t \in R$