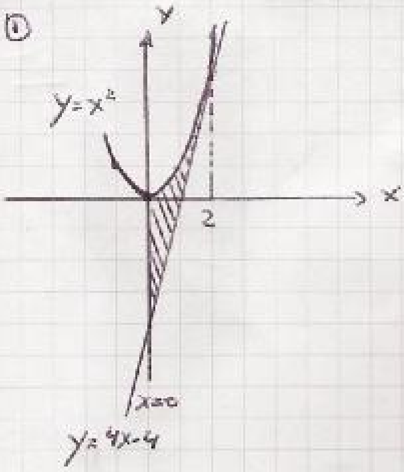
PARCIAL 1:

Michelini Curso de Verano 2004 PARCIAL 1

①



Ptos de intersección

$x^2 = 4x-4$

$x^2-4x+4=0$

$\frac{4 \pm \sqrt{16-16}}{2} = 2 = x_1 = x_2$

$$Area = \int_0^2 x^2-(4x-4)\,dx = \frac{x^3}{3} - \frac{4x^2}{2} + 4x \Big|_0^2 = \boxed{\frac{8}{3}}$$

② $$\int_1^{+\infty} \frac{1}{x(\ln x)^2}\,dx = \lim_{b\to+\infty} \int_1^b \frac{1}{x}(\ln x)^{-2}\,dx = \lim_{b\to+\infty} -\frac{1}{\ln(x)}\Big|_1^b = \lim_{b\to+\infty} \left(\frac{-1}{\ln(b)}\right) + \frac{1}{0} =$$

$=$ No existe (indefinido)

C. Auxiliar

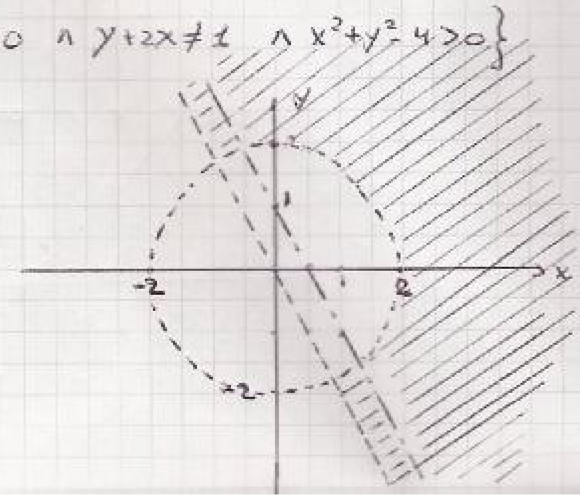
$$\int \frac{1}{x}(\ln x)^{-2}\,dx = \int z^{-2}\,dz = \frac{z^{-1}}{-1}+c = -\frac{1}{z}+c = \boxed{\frac{-1}{\ln(x)}+c}$$

$z = \ln(x)$

$dz = \frac{1}{x}\,dx$

③ $$\vec{\nabla} f(x,y) = \left(\frac{1}{xy}.y + y^2(-\operatorname{sen}(xy)).y + e^{xy}.y \; ; \; \frac{1}{xy}x + 2y\cos(xy) - y^2\operatorname{sen}(xy)x + e^{xy}x\right)$$

④ $Dm f : \{(x,y)\in \mathbb{R}^2 \,/\, y+2x>0 \;\wedge\; y+2x \neq 1 \;\wedge\; x^2+y^2-4>0\}$



PARCIAL 2:

Micheloni Curso de Verano 2004

① $\int x^{-2} e^{\frac{2}{x}}\, dx$

$z = \frac{2}{x}$

$dz = -\frac{2}{x^2}\, dx \quad \Rightarrow \quad \int e^{z} \frac{dz}{-2} = -\frac{1}{2}\int e^{z} dz = -\frac{1}{2} e^{z} + c = \boxed{-\frac{1}{2} e^{\frac{2}{x}} + c}$

$\frac{dz}{-2} = x^{-2} dx$

② $\int_1^{+\infty} x\sqrt{1+3x^2}\, dx = \lim_{b\to+\infty} \int_1^{b} x\,(1+3x^2)^{1/2}\, dx = \lim_{b\to+\infty} \frac{1}{9}\sqrt{(1+3x^2)^3}\,\Big|_1^b =$

$= \lim_{b\to+\infty} \frac{1}{9}\underbrace{\sqrt{(1+3b^2)^3}}_{+\infty} - \frac{1}{9}\sqrt{64} = \boxed{+\infty}$

Divergente

c. Auxiliar

$\int x\,(1+3x^2)^{1/2}\, dx$

$z = 1+3x^2$

$dz = 6x\, dx \quad \Rightarrow \quad \int z^{1/2} \frac{dz}{6} = \frac{1}{6}\int z^{1/2} dz = \frac{1}{6}\,\frac{z^{3/2}}{3/2} + c = \frac{1}{9}\sqrt{(1+3x^2)^3} + c$

$\frac{dz}{6} = dx$

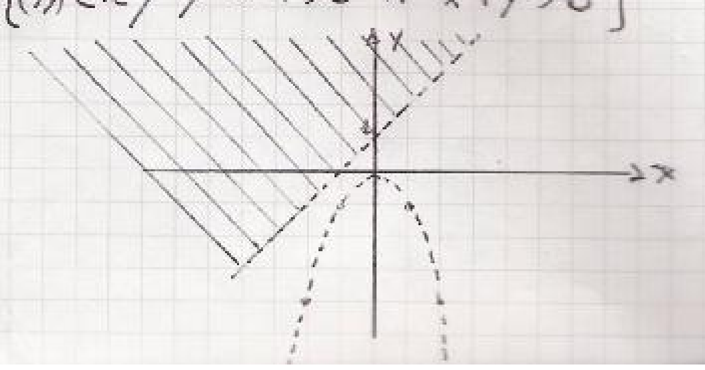
③ $f'_x(x,y) = e^{\frac{x}{y}}\,\frac{1}{y}\,\ln\left(\frac{x^2}{y}\right) + e^{x/y}\,\frac{y}{x^2}\cdot\frac{2x}{y} = \Big|\; e^2\left(\ln(4)+1\right)$

$f'_y(x,y) = e^{\frac{x}{y}}\left(\frac{-x}{y^2}\right)\ln\left(\frac{x^2}{y}\right) + e^{x/y}\,\frac{y}{x^2}\cdot\left(\frac{-x^2}{y^2}\right) = \Big|\; e^2\left(-2\ln(4)-1\right)$

$P = (2,1)$

$\Rightarrow \vec{\nabla} f(2,1) = \left(e^2(\ln 4+1)\,;\; e^2(-2\ln 4-1)\right)$

④ $\text{Dom } z = \left\{(x,y)\in\mathbb{R}^2 \,/\, y-x-1>0 \;\wedge\; x^2+y>0\right\}$



PARCIAL 3:

Micheloni 1º cuatrimestre 2004

1) $\int_{-\infty}^{2} e^{-2x+1}\,dx = \lim_{a\to-\infty} \int_a^2 e^{-2x+1}\,dx = \lim_{a\to-\infty} -\frac{1}{2} e^{-2x+1}\Big|_a^2 =$

$= \lim_{a\to-\infty} -\frac{1}{2} e^{-3} + \frac{1}{2} e^{-2a+1} = \boxed{\infty}$ Diverge

C. auxiliar

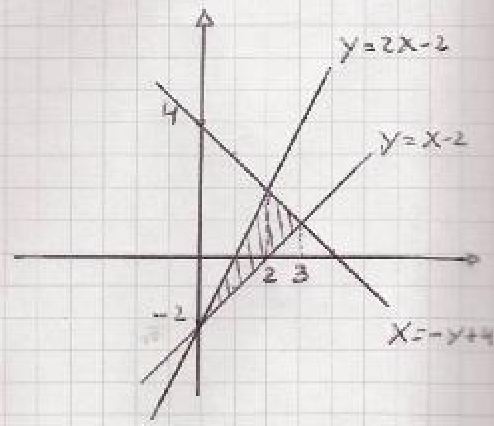
$z = -2x+1$

$dz = -2 \cdot dx$

$\frac{dz}{-2} = dx$

$\int e^z \frac{dz}{-2} = -\frac{1}{2} e^z + c = \boxed{-\frac{1}{2} e^{-2x+1} + c}$

2) $y = 2x-2$; $y = x-2$; $x = -y+4$



$2x-2 = -x+4$
$3x = 6$
$\boxed{x=2}$

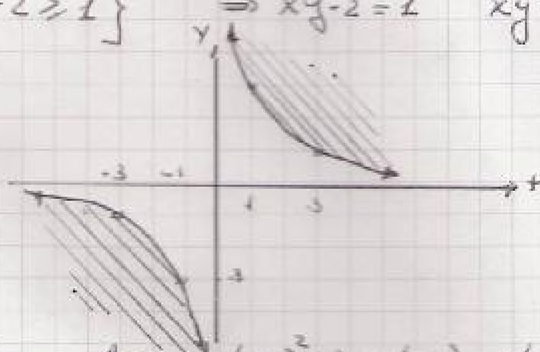
$x-2 = -x+4$
$2x = 6$
$\boxed{x=3}$

$A_1 = \int_0^2 (2x-2)-(x-2)\,dx = \int_0^2 x\,dx = \frac{x^2}{2}\Big|_0^2 = \boxed{2}$

$A_2 = \int_2^3 (-x+4)-(x-2)\,dx = \int_2^3 6-2x\,dx = 6x-x^2\Big|_2^3 = 9-8 = \boxed{1}$

$\Rightarrow A = A_1 + A_2 = \boxed{3}$

3) $\text{Dom} f = \{(x,y) \in \mathbb{R}^2 / \ xy-2 \geq 1\}$ $\Rightarrow xy-2 = 1$ $\quad xy = 3$ $\quad \boxed{y = \frac{3}{x}}$



4) $z'_x(2,-1) = \lim_{h\to0} \frac{F(2+h,-1)-F(2,-1)}{h} = \lim_{h\to0} \frac{(2+h)^2(-1)+3(2+h)-1(2+h)^2-(-2)}{h} =$

$= \lim_{h\to0} \frac{\cancel{-4}-4h-h^2+\cancel{6}+3h\cancel{-4}-4h-h^2+\cancel{2}}{h} = \lim_{h\to0} \frac{\cancel{h}(-8+3-2h)}{\cancel{h}} = \boxed{-5}$

PARCIAL 4:

Angeles 2do Cuatrimestre 2004

1)

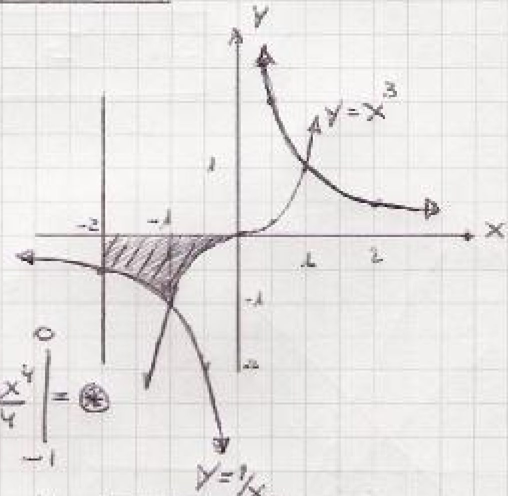
$$f(x) = \int f'(x)\,dx = \int 5x^4 \cos(3x^5)\,dx$$

$$z = 3x^5$$
$$dz = 15x^4\,dx$$
$$\frac{dz}{3} = 5x^4\,dx \quad \Rightarrow \int \cos(z)\,\frac{dz}{3} = \frac{1}{3}\,\mathrm{Sen}(z) + c = \boxed{\frac{1}{3}\,\mathrm{Sen}(3x^5) + c}$$

2) $y = x^3$; $y = \frac{1}{x}$; $x = -2$ ∧ eje x

$$\frac{1}{x} = x^3 \Rightarrow 1 = x^4 \Rightarrow 1 = |x|$$

$$\boxed{\begin{array}{l} x = 1 \\ x = -1 \end{array}}$$



$$A = \int_{-2}^{-1} 0 - \frac{1}{x}\,dx + \int_{-1}^{0} 0 - x^3\,dx = -\ln|x|\,\Big|_{-2}^{-1} + -\frac{x^4}{4}\Big|_{-1}^{0} = ⊛$$

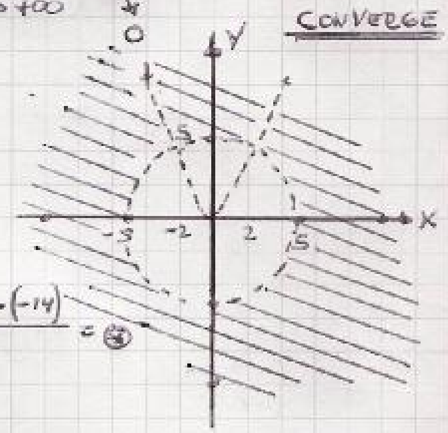
$$⊛ = -\ln|-1| + \ln|-2| + \frac{(-1)^4}{4} = \boxed{\frac{1}{4} + \ln(2)}$$

3)

$$\int_1^{+\infty} e^{-x}\,dx = \lim_{b \to +\infty} \int_1^{b} e^{-x}\,dx = \lim_{b \to +\infty} -e^{-x}\Big|_1^{b} = \lim_{b \to +\infty} -\underbrace{e^{-b}}_{\to 0} + e^{-1} = \boxed{e^{-1}}$$

CONVERGE

4) $\mathrm{Dom}\,f = \{(x,y) \in \mathbb{R}^2 / \ x^2 + y^2 - 25 > 0 \ \wedge \ y - x^2 \neq 0\}$



5)

$$f'_x(1;2) = \lim_{h \to 0} \frac{f(1+h;2) - f(1;2)}{h} = \lim_{h \to 0} \frac{2(1+h) - 4(1+h)^2 \cdot 4 - (-14)}{h} = ⊛$$

$$⊛ = \lim_{h \to 0} \frac{2 + 2h - (4 + 8h + 4h^2)4 + 14}{h} =$$

$$= \lim_{h \to 0} \frac{\cancel{2} - \cancel{16} - 32h - 16h^2 + \cancel{14} + 2h}{h} = \lim_{h \to 0} \frac{\cancel{h}(-30 - \overbrace{16h}^{\to 0})}{\cancel{h}} = \boxed{-30}$$

6) $f(x,y) = e^{3xy} - 2\ln(y^2x)$

a)
$$f'_x = e^{3xy}\,3y - \frac{2}{\cancel{y^2}x}\,\cancel{y^2} = 6e^6 - 2$$
$$f'_y = e^{3xy}\,3x - \frac{2}{y^{\cancel{2}}\cancel{x}} \cdot 2y\cancel{x} = 3e^6 - 2$$
$$P_0 = (1;2)$$

$$\Rightarrow \nabla f(1;2) = (6e^6 - 2 \,;\, 3e^6 - 2)$$

b) $f''_{xy} = e^{3xy}\,3x\,3y + e^{3xy}\,3$ ✓ $\qquad f''_{yx} = e^{3xy} \cdot 3y \cdot 3x + e^{3xy} \cdot 3$ ✓

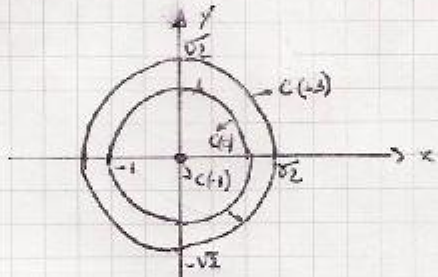
PARCIAL 5:

Micheloni 2do cuatrimestre 2004

1) $C(0) = \{(x,y) \in \text{Dom } z \,/\, x^2+y^2-1=0\} \Rightarrow x^2+y^2=1$

$C(-1) = \{ \text{"} \;\; \text{"} \;\; \text{"} \,/\, x^2+y^2-1=-1\} \Rightarrow x^2+y^2=0$

$C(-3) = \{ \text{"} \;\; \text{"} \;\; \text{"} \,/\, x^2+y^2-1=-3\} \Rightarrow x^2+y^2=2$



2) $$\int_{-\infty}^{-1} 2e^x\sqrt{1-e^x}\,dx = \lim_{a\to-\infty}\int_a^{-1} 2e^x\sqrt{1-e^x}\,dx = \lim_{a\to-\infty} -\frac{4}{3}\sqrt{(1-e^x)^3}\Big|_a^{-1} =$$

$$= \lim_{a\to-\infty} -\frac{4}{3}\sqrt{(1-e^{-1})^3}+\frac{4}{3}\underbrace{\sqrt{(1-e^a)^3}}_{\to 1} = \boxed{-\frac{4}{3}\sqrt{(1-e^{-1})^3}+\frac{4}{3}}$$

converge

C Aux

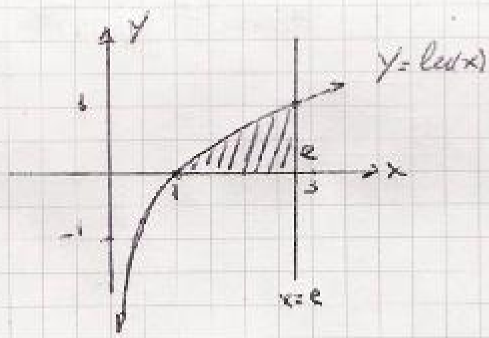
$z = 1-e^x$

$dz = -e^x dx$

$-dz = e^x dx$

$$2\int z^{1/2}(-dz) = -2\,\frac{z^{3/2}}{3/2}+c = -\frac{4}{3}\sqrt{(1-e^x)^3}+c$$

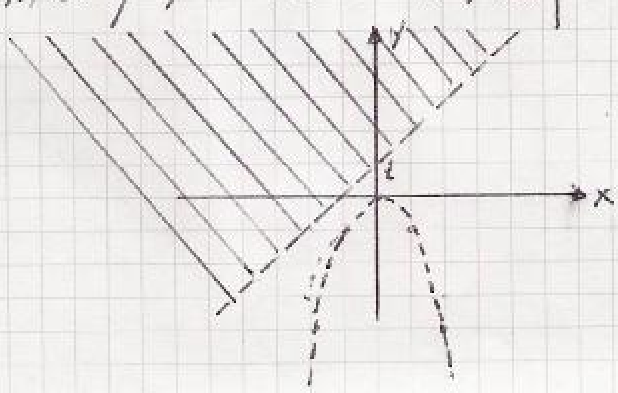
3) $y=\ln(x)$; $y=0$, $x=e$



$$A = \int_1^e \ln(x) - 0\,dx = \ln(x)\,x - x\Big|_1^e =$$

$$= (\ln(e)\,e-e)-(\ln(1)\,1-1) = \boxed{1}$$

4) $\text{Dom } z = \{(x,y)\in \mathbb{R}^2 \,/\, y-x-1>0 \;\wedge\; x^2+y>0\}$



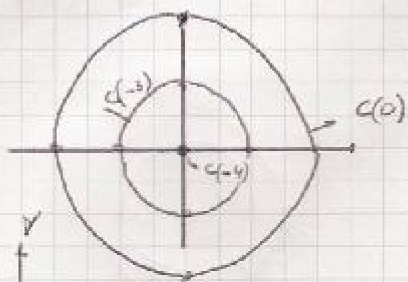
PARCIAL 6:

[illegible] 2 - cuatrimestre 2004

1) $C(0) = \{(x,y) \in Dom\, z \,/\, x^2+y^2-4=0\} \Rightarrow x^2+y^2=4$

$C(-4) = \{ \text{"} \quad \text{"} \quad \text{"} \,/\, x^2+y^2-4=-4\} \Rightarrow x^2+y^2=0$

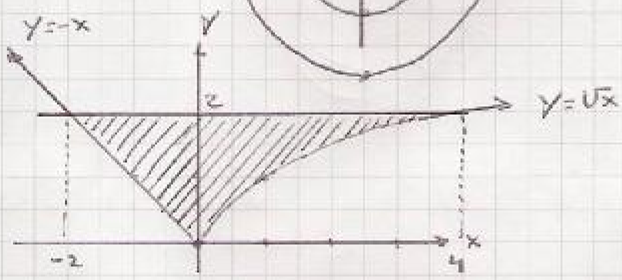
$C(-3) = \{ \text{"} \quad \text{"} \quad \text{"} \,/\, x^2+y^2-4=-3\} \Rightarrow x^2+y^2=1$



2) $y=-x$

$y=\sqrt{x}$

$y=2$



$-x=2 \Rightarrow x=-2$

$\sqrt{x}=2 \Rightarrow x=4$

$$\Rightarrow A = \int_{-2}^{0} 2-(-x)\,dx + \int_{0}^{4} 2-\sqrt{x}\,dx = 2x+\frac{x^2}{2}\Big|_{-2}^{0} + 2x - \frac{2}{3}\sqrt{x^3}\Big|_{0}^{4} = -(-4+2)+\left(8-\frac{16}{3}\right) = 2+\frac{8}{3} = \boxed{\frac{14}{3}}$$

3) $$\int_{-2}^{+\infty} 2e^{1/x}x^{-2}\,dx = \lim_{b\to+\infty}\int_{-2}^{b} 2e^{1/x}x^{-2}\,dx = \lim_{b\to+\infty} -2e^{1/x}\Big|_{-2}^{b} = *$$

$$* = \lim_{b\to+\infty} -2\underbrace{e^{1/b}}_{1} + 2e^{-1/2} = \boxed{-2+\frac{2}{\sqrt{e}}}$$

converge

C aux

$z = 1/x$

$dz = -x^{-2}dx$

$\frac{dz}{-1} = x^{-2}dx$

$$\int 2e^{z}(-dz) = -2e^{z}+c = -2e^{1/x}$$

4) $Dom\, z = \{(x,y)\in \mathbb{R}^2 \,/\, 4x^2-y+2 \geq 0 \wedge x^2+y>0 \wedge x^2+y \neq 1\}$